ISSN 2181-8622

Manufacturing technology problems

Scientific and Technical Journal Namangan Institute of Engineering and Technology

Volume 10
Issue 4
2025

UDC 519.6

DERIVATION OF DIFFERENTIAL EQUATIONS FOR SPINDLE OSCILLATION IN A SYSTEM OF RECTANGULAR COORDINATES

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Abstract: This article develops mathematical models for the deformation of structural elements such as rods under spatially dynamic loading. The resulting vibration equations are described by systems of second-order partial differential equations with natural boundary and initial conditions.

Keywords: displacement, deformation, variational principle, variations in kinetic and potential energies, variation in the work of an external force, mathematical model, boundary value problem, system of partial differential equations, boundary and initial conditions.

Introduction. In many developed countries, special attention is being paid to mathematical modeling and numerical analysis of structures and constructions. Structural elements such as rods are widely used in many areas of engineering, mechanical engineering, and construction. This necessitates testing their strength, deformability, and stability using mathematical models. Leading countries, including the United States, Japan, Germany, Italy, Russia, and others, are focusing on developing mathematical modeling and calculation methods for construction and structural materials.

Improving mathematical models and computational algorithms for solving linear and nonlinear problems of structural materials is relevant on a global scale.

Since independence, the republic's mechanical engineering and construction industries have developed, and the requirements for the design process of structures and buildings have increased. Determining and assessing the stress-strain state of structural materials, with the extensive use of modern computer technology, are considered the most important tasks of design and survey work.

Derivation of differential equations for spindle oscillations

Based on the assumptions given in the work of V.K. Kabulov [1], the expressions for the displacement of the rod points during longitudinal, transverse and torsional vibrations are presented in the form.

$$u_1 = u - z\alpha_1 - y\alpha_2,\quad u_2 = v + z\theta,\quad u_3 = w - y\theta \tag{1}$$

where u, v, w are the displacements of the rod center line; α_1, α_2 are the angles of inclination of the tangent to the elastic line under pure bending; θ is the twist angle;

Now, according to the Cauchy formula, taking into account formulas (1), we calculate the deformation components [1,2]:

$$\varepsilon_{11}=\frac{du}{dx}-z\frac{d\alpha_1}{dx}-y\frac{d\alpha_2}{dx},\ \ \varepsilon_{12}=\frac{dv}{dx}+z\frac{d\theta}{dx}-\alpha_2,$$

$$\varepsilon_{13}=\frac{dw}{dx}-y\frac{d\theta}{dx}-\alpha_1,\ \ \varepsilon_{22}=\varepsilon_{33}=\varepsilon_{23}=0. \tag{2}$$

The stress and strain components are related as follows:

$$\sigma_{11}=E\left(\frac{\partial u}{\partial x}-z\frac{\partial\alpha_1}{\partial x}-y\frac{\partial\alpha_2}{\partial x}\right),\ \ \sigma_{12}=G\left(\frac{\partial v}{\partial x}+z\frac{\partial\theta}{\partial x}-\alpha_2\right),$$

$$\sigma_{13}=G\left(\frac{\partial w}{\partial x}-y\frac{\partial\theta}{\partial x}-\alpha_1\right),\ \ \sigma_{22}=\sigma_{33}=\sigma_{23}=0. \tag{3}$$

In this case, variations in kinetic energy can be represented as.

$$\int_t \delta \mathrm{T}dt=\int_t\int_V\left[\rho\frac{\partial u_1}{\partial t}\delta\frac{\partial u_1}{\partial t}+\rho\frac{\partial u_2}{\partial t}\delta\frac{\partial u_2}{\partial t}+\rho\frac{\partial u_3}{\partial t}\delta\frac{\partial u_3}{\partial t}\right]dVdt \tag{4}$$

Here we perform integration by parts operations

$$\int_t \delta \mathrm{T}dt=\int_V\left[\rho\frac{\partial u_1}{\partial t}\delta u_1+\rho\frac{\partial u_2}{\partial t}\delta u_2+\rho\frac{\partial u_3}{\partial t}\delta u_3\right]dV\Big|_t-$$
$$-\int_t\int_V\left[\rho\frac{\partial^2 u_1}{\partial t^2}\delta u_1+\rho\frac{\partial^2 u_2}{\partial t^2}\delta u_2+\rho\frac{\partial^2 u_3}{\partial t^2}\delta u_3\right]dVdt \tag{5}$$

Substituting the expressions u_i from (1) into the variations of kinetic energy (5) we obtain

$$\int_t d\mathrm{T}dt=\int_V\left[\rho\frac{\partial u_1}{\partial t}\delta(u-z\alpha_1-y\alpha_2)+\rho\frac{\partial u_2}{\partial t}\delta(v+z\theta)+\rho\frac{\partial u_3}{\partial t}\delta(w-y\theta)\right]dV\Big|_t-$$
$$-\int_t\int_V\left[\rho\frac{\partial^2 u_1}{\partial t^2}\delta(u-z\alpha_1-y\alpha_2)+\rho\frac{\partial^2 u_2}{\partial t^2}\delta(v+z\theta)+\rho\frac{\partial^2 u_3}{\partial t^2}\delta(w-y\theta)\right]dVdt=$$
$$=\int_V\left[\rho\frac{\partial}{\partial t}(u-z\alpha_1-y\alpha_2)\delta u-\rho z\frac{\partial}{\partial t}(u-z\alpha_1-y\alpha_2)\delta\alpha_1-\right.$$
$$-\rho y\frac{\partial}{\partial t}(u-z\alpha_1-y\alpha_2)\delta\alpha_2+\rho\frac{\partial}{\partial t}(v+z\theta)\delta v+$$
$$\left.+\rho z\frac{\partial}{\partial t}(v+z\theta)\delta\theta+\rho\frac{\partial}{\partial t}(w-y\theta)\delta w-\rho y\frac{\partial}{\partial t}(w-y\theta)\delta\theta\right]dV\Big|_t-$$
$$-\int_t\int_V\left[\rho\frac{\partial^2}{\partial t^2}(u-z\alpha_1-y\alpha_2)\delta u-\rho z\frac{\partial^2}{\partial t^2}(u-z\alpha_1-y\alpha_2)\delta\alpha_1-\right.$$
$$-\rho y\frac{\partial^2}{\partial t^2}(u-z\alpha_1-y\alpha_2)\delta\alpha_2+\rho\frac{\partial^2}{\partial t^2}(v+z\theta)\delta v+\rho z\frac{\partial^2}{\partial t^2}(v+z\theta)\delta\theta+$$
$$\left.+\rho\frac{\partial^2}{\partial t^2}(w-y\theta)\delta w-\rho y\frac{\partial^2}{\partial t^2}(w-y\theta)\delta\theta\right]dVdt$$

Let us extract the integral over the cross-sections of the rod

$$\int_t d\mathrm{T}dt = \int_x \left\{ \left[\rho \int_F dF \frac{\partial u}{\partial t} - \rho \int_F zdF \frac{\partial \alpha_1}{\partial t} - \rho \int_F ydF \frac{\partial \alpha_2}{\partial t} \right] \delta u - \right.$$

$$- \left[\rho \int_F zdF \frac{\partial u}{\partial t} - \rho \int_F z^2 dF \frac{\partial \alpha_1}{\partial t} - \rho \int_F zydF \frac{\partial \alpha_2}{\partial t} \right] \delta \alpha_1 -$$

$$- \left[\rho \int_F ydF \frac{\partial u}{\partial t} - \rho \int_F yzdF \frac{\partial \alpha_1}{\partial t} - \rho \int_F y^2 dF \frac{\partial \alpha_2}{\partial t} \right] \delta \alpha_2 +$$

$$+ \left[\rho \int_F dF \frac{\partial v}{\partial t} + \rho \int_F zdF \frac{\partial \theta}{\partial t} \right] \delta v + \left[\rho \int_F zdF \frac{\partial v}{\partial t} + \rho \int_F z^2 dF \frac{\partial \theta}{\partial t} \right] \delta \theta +$$

$$\left. + \left[\rho \int_F dF \frac{\partial w}{\partial t} - \rho \int_F ydF \frac{\partial \theta}{\partial t} \right] \delta W - \left[\rho \int_F ydF \frac{\partial w}{\partial t} - \rho \int_F y^2 dF \frac{\partial \theta}{\partial t} \right] \delta \theta \right\} dx\Big|_t -$$

$$- \int_t \int_x \left\{ \left[\rho \int_F dF \frac{\partial^2 u}{\partial t^2} - \rho \int_F zdF \frac{\partial^2 \alpha_1}{\partial t^2} - \rho \int_F ydF \frac{\partial^2 \alpha_2}{\partial t^2} \right] \delta u - \right.$$

$$- \left[\rho \int_F zdF \frac{\partial^2 u}{\partial t^2} - \rho \int_F z^2 dF \frac{\partial^2 \alpha_1}{\partial t^2} - \rho \int_F zydF \frac{\partial^2 \alpha_2}{\partial t^2} \right] \delta \alpha_1 -$$

$$- \left[\rho \int_F ydF \frac{\partial^2 u}{\partial t^2} - \rho \int_F yzdF \frac{\partial^2 \alpha_1}{\partial t^2} - \rho \int_F y^2 dF \frac{\partial^2 \alpha_2}{\partial t^2} \right] \delta \alpha_2 +$$

$$+ \left[\rho \int_F dF \frac{\partial^2 v}{\partial t^2} + \rho \int_F zdF \frac{\partial^2 \theta}{\partial t^2} \right] \delta v + \left[\rho \int_F zdF \frac{\partial^2 v}{\partial t^2} + \rho \int_F z^2 dF \frac{\partial^2 \theta}{\partial t^2} \right] \delta \theta +$$

$$\left. + \left[\rho \int_F dF \frac{\partial^2 w}{\partial t^2} - \rho \int_F ydF \frac{\partial^2 \theta}{\partial t^2} \right] \delta W - \left[\rho \int_F ydF \frac{\partial^2 w}{\partial t^2} - \rho \int_F y^2 dF \frac{\partial^2 \theta}{\partial t^2} \right] \delta \theta \right\} dxdt$$

Let's introduce the following notations:

$$F = \int_F dF, \; S_z = \int_F ydF, \; I_y = \int_F z^2 dF, \; I_{yz} = \int_F yzdF,$$

$$S_y = \int_F zdF, \; I_z = \int_F y^2 dF. \tag{6}$$

Taking into account the introduced notations (6), we rewrite the variations of kinetic energy

$$\int_t dTdt = \int_x \left\{ \left[\rho F \frac{\partial u}{\partial t} - \rho S_y \frac{\partial \alpha_1}{\partial t} - \rho S_z \frac{\partial \alpha_2}{\partial t} \right] \delta u - \right.$$

$$- \left[\rho S_y \frac{\partial u}{\partial t} - \rho I_y \frac{\partial \alpha_1}{\partial t} - \rho I_{yz} \frac{\partial \alpha_2}{\partial t} \right] \delta \alpha_1 -$$

$$- \left[\rho S_z \frac{\partial u}{\partial t} - \rho I_{yz} \frac{\partial \alpha_1}{\partial t} - \rho I_z \frac{\partial \alpha_2}{\partial t} \right] \delta \alpha_2 +$$

$$+ \left[\rho F \frac{\partial v}{\partial t} + \rho S_y \frac{\partial \theta}{\partial t} \right] \delta v + \left[\rho S_y \frac{\partial v}{\partial t} + \rho I_y \frac{\partial \theta}{\partial t} \right] \delta \theta +$$

$$\left. + \left[\rho F \frac{\partial w}{\partial t} - \rho S_z \frac{\partial \theta}{\partial t} \right] \delta W - \left[\rho S_z \frac{\partial w}{\partial t} - \rho I_z \frac{\partial \theta}{\partial t} \right] \delta \theta \right\} dx \Big|_t -$$

$$- \int_t \int_x \left\{ \left[\rho F \frac{\partial^2 u}{\partial t^2} - \rho S_y \frac{\partial^2 \alpha_1}{\partial t^2} - \rho S_z \frac{\partial^2 \alpha_2}{\partial t^2} \right] \delta u - \right.$$

$$- \left[\rho S_y \frac{\partial^2 u}{\partial t^2} - \rho I_y \frac{\partial^2 \alpha_1}{\partial t^2} - \rho I_{yz} \frac{\partial^2 \alpha_2}{\partial t^2} \right] \delta \alpha_1 -$$

$$- \left[\rho S_z \frac{\partial^2 u}{\partial t^2} - \rho I_{yz} \frac{\partial^2 \alpha_1}{\partial t^2} - \rho I_z \frac{\partial^2 \alpha_2}{\partial t^2} \right] \delta \alpha_2 +$$

$$+ \left[\rho F \frac{\partial^2 v}{\partial t^2} + \rho S_y \frac{\partial^2 \theta}{\partial t^2} \right] \delta v + \left[\rho S_y \frac{\partial^2 v}{\partial t^2} + \rho I_y \frac{\partial^2 \theta}{\partial t^2} \right] \delta \theta +$$

$$\left. + \left[\rho F \frac{\partial^2 w}{\partial t^2} - \rho S_z \frac{\partial^2 \theta}{\partial t^2} \right] \delta W - \left[\rho S_z \frac{\partial^2 w}{\partial t^2} - \rho I_z \frac{\partial^2 \theta}{\partial t^2} \right] \delta \theta \right\} dxdt \quad (7)$$

Now let us consider the variations of potential energy $\delta\Pi$:

$$\int_t \delta \Pi dt = \int_t \int_V \left(\sum_{j=1}^{3} \sigma_{1j} \delta \varepsilon_{1j} \right) dVdt \quad (8)$$

Substituting the expressions u_i from (1) into the variations of potential energy (8) we obtain

$$\int_t \delta \Pi dt = \int_t \int_V \left[\sigma_{11} \delta \left(\frac{\partial u}{\partial x} - z \frac{\partial \alpha_1}{\partial x} - y \frac{\partial \alpha_2}{\partial x} \right) + \right.$$

$$\left. + \sigma_{12} \delta \left(\frac{\partial v}{\partial x} + z \frac{\partial \theta}{\partial x} - \alpha_2 \right) + \sigma_{13} \delta \left(\frac{\partial w}{\partial x} - y \frac{\partial \theta}{\partial x} - \alpha_1 \right) \right] dVdt$$

Let's perform integration by parts. Before doing this, let's extract the integral over the cross-section of the rod.

$$\int_t \delta\Pi dt = \int_t\int_x \Bigg[\int_F \sigma_{11} dF \delta \frac{\partial u}{\partial x} - \int_F z\sigma_{11} dF \delta \frac{\partial \alpha_1}{\partial x} - \int_F y\sigma_{11} dF \delta \frac{\partial \alpha_2}{\partial x} +$$

$$+ \int_F \sigma_{12} dF \delta \frac{\partial v}{\partial x} + \int_F z\sigma_{12} dF \delta \frac{\partial \theta}{\partial x} - \int_F \sigma_{12} dF \delta\alpha_2 +$$

$$+ \int_F \sigma_{13} dF \delta \frac{\partial w}{\partial x} - \int_F y\sigma_{13} dF \delta \frac{\partial \theta}{\partial x} - \int_F \sigma_{13} dF \delta\alpha_1 \Bigg] dxdt =$$

$$= \int_t \left[N_x \delta u - M_y \delta\alpha_1 - M_z \delta\alpha_2 + Q_{12}\delta v + M_x \delta\theta + Q_{13}\delta W \right] dt\Big|_x -$$

$$- \int_t\int_x \Bigg[\frac{\partial N_x}{\partial x}\delta u - \frac{\partial M_y}{\partial x}\delta\alpha_1 - \frac{\partial M_z}{\partial x}\delta\alpha_2 + \frac{\partial Q_{12}}{\partial x}\delta v +$$

$$+ \frac{\partial M_x}{\partial x}\delta\theta + \frac{\partial Q_{13}}{\partial x}\delta w + Q_{12}\delta\alpha_2 + Q_{13}\delta\alpha_1 \Bigg] dxdt \qquad (9)$$

The following notations are introduced here:

$$N_x = \int_F \sigma_{11} dF,\; M_y = \int_F z\sigma_{11} dF,\; M_z = \int_F y\sigma_{11} dF,\; Q_{12} = \int_F \sigma_{12} dF,$$

$$M_x = \int_F (z\sigma_{12} - y\sigma_{13}) dF \quad Q_{13} = \int_F \sigma_{13} dF \qquad (10)$$

Now let's consider variations of the work of external forces δA.

$$\int_t \delta A dt = \int_V \sum_{i=1}^{3} P_i \delta u_i dV + \int_S \sum_{i=1}^{3} q_i \delta u_i dS + \int_{S_1} \sum_{i=1}^{3} \phi_i \,\delta u_i dS_1\Big|_x =$$

$$= \int_t\int_V \left[P_1\delta u_1 + P_2\delta u_2 + P_3\delta u_3 \right] dVdt + \qquad (11)$$

$$+ \int_t\int_S [q_1\delta u_1 + q_2\delta u_2 + q_3\delta u_3] dSdt + \int_t\int_{S_1} \left[\phi_1\delta u_1 + \phi_2\delta u_2 + \phi_3\delta u_3 \right] dS_1 dt\Big|_x$$

Here we will extract the integral over the cross-section of the rod using relation (1).

$$\int_t \delta A dt = \int_t\int_x \Bigg[\int_F P_1 dF \delta u - \int_F zP_1 dF \delta\alpha_1 - \int_F yP_1 dF \delta\alpha_2 + \int_F P_2 dF \delta v +$$

$$+ \int_F zP_2 dF \delta\theta + \int_F P_3 dF \delta w - \int_F yP_3 dF \delta\theta \Bigg] dxdt +$$

$$+ \int_t\int_x \Bigg[\int_l q_1 dl \delta u - \int_l zq_1 dl \delta\alpha_1 - \int_l yq_1 dl \delta\alpha_2 + \int_l q_2 dl \delta v +$$

$$+ \int_l zq_2 dl \delta\theta + \int_l q_3 dl \delta w - \int_l yq_3 dl \delta\theta \Bigg] dxdt$$

$$+ \int_t\int_{S_1} \left[\phi_1\delta u - z\phi_1\delta\alpha_1 - y\phi_1\delta\alpha_2 + \phi_2\delta v + z\phi_2\delta\theta + \phi_3\delta w - y\phi_3\delta\theta \right] dS_1 dt\Big|_x$$

The following notations are introduced here:

$$N_x(P_1)=\int_F P_1 dF,\quad M_y(P_1)=\int_F zP_1 dF,\quad M_z(P_1)=\int_F yP_1 dF,\quad Q_{12}(P_2)=\int_F P_2 dF,$$

$$M_x(P_2,P_3)=\int_F (zP_2-yP_3)dF,\qquad Q_{13}(P_3)=\int_F P_3 dF;$$

$$N_x(q_1)=\int_l q_1 dl,\quad M_y(q_1)=\int_l zq_1 dl,\quad M_z(q_1)=\int_l yq_1 dl,\quad Q_{12}(q_2)=\int_l q_2 dl,$$

$$M_x(q_2,q_3)=\int_l (zq_2-yq_3)dl,\qquad Q_{13}(q_3)=\int_l q_3 dl;$$

$$N_x(\phi_1)=\int_{S_1}\phi_1 dS_1,\quad M_y(\phi_1)=\int_{S_1} z\phi_1 dS_1,\quad M_z(\phi_1)=\int_{S_1} y\phi_1 dS_1,\quad Q_{12}(\phi_2)=\int_{S_1}\phi_2 dS_1,$$

$$M_x(\phi_2,\phi_3)=\int_{S_1}(z\phi_2-y\phi_3)dS_1,\qquad Q_{13}(\phi_3)=\int_{S_1}\phi_3 dS_1; \tag{12}$$

Taking into account the introduced notations (12), we rewrite the variations of the work of external forces δA:

$$\begin{aligned}\int_t \delta A dt=\int_t\Bigg[\int_x\Big[&\big(N_x(P_1)+N_x(q_1)\big)\delta u-\big(M_y(P_1)+M_y(q_1)\big)\delta\alpha_1-\\&-\big(M_z(P_1)+M_z(q_1)\big)\delta\alpha_2+\big(Q_{12}(P_2)+Q_{12}(q_2)\big)\delta v+\\&+\big(M_x(P_2,P_3)+M_x(q_2,q_3)\big)\delta\theta+\big(Q_{13}(P_3)+Q_{13}(q_3)\big)\delta w\Big]dx+\\&+\Big[\big(N_x(\phi_1)\delta u-M_y(\phi_1)\delta\alpha_1-M_z(\phi_1)\delta\alpha_2+\\&+Q_{12}(\phi_2)\delta v+M_x(\phi_2,\phi_3)\delta\theta+Q_{13}(\phi_3)\delta w\Big]\Big|_x\Bigg]dt\end{aligned} \tag{13}$$

We substitute variations of kinetic (7), potential (9) energy and work of external forces (13) into the Hamilton-Ostrogradsky variational principle

$$\int_t(\delta T-\delta\Pi+\delta A)dt=0 \tag{14}$$

$$\begin{aligned}&\int_t(\delta T-\delta\Pi+\delta A)dt=\\&=\int_t\int_x\Bigg\{\left[-\rho F\frac{\partial^2 u}{\partial t^2}+\rho S_y\frac{\partial^2\alpha_1}{\partial t^2}+\rho S_z\frac{\partial^2\alpha_2}{\partial t^2}+\frac{\partial N_x}{\partial x}+\big(N_x(P_1)+N_x(q_1)\big)\right]\delta u+\\&+\left[\rho S_y\frac{\partial^2 u}{\partial t^2}-\rho I_y\frac{\partial^2\alpha_1}{\partial t^2}-\rho I_{yz}\frac{\partial^2\alpha_2}{\partial t^2}-\frac{\partial M_y}{\partial x}+Q_{13}-\big(M_y(P_1)+M_y(q_1)\big)\right]\delta\alpha_1+\\&+\left[\rho S_z\frac{\partial^2 u}{\partial t^2}-\rho I_{yz}\frac{\partial^2\alpha_1}{\partial t^2}-\rho I_z\frac{\partial^2\alpha_2}{\partial t^2}-\frac{\partial M_z}{\partial x}+Q_{12}-\big(M_z(P_1)+M_z(q_1)\big)\right]\delta\alpha_2+\end{aligned}$$

$$+\left[-\rho F\frac{\partial^2 v}{\partial t^2}-\rho S_y\frac{\partial^2\theta}{\partial t^2}+\frac{\partial Q_{12}}{\partial x}+\left(Q_{12}(P_2)+Q_{12}(q_2)\right)\right]\delta v+$$

$$+\left[-\rho S_y\frac{\partial^2 v}{\partial t^2}-\rho\left(I_y+I_z\right)\frac{\partial^2\theta}{\partial t^2}+\rho S_z\frac{\partial^2 w}{\partial t^2}+\frac{\partial M_x}{\partial x}+\left(M_x(P_2,P_3)+M_x(q_2,q_3)\right)\right]\delta\theta+$$

$$+\left.\left[-\rho F\frac{\partial^2 w}{\partial t^2}+\rho S_z\frac{\partial^2\theta}{\partial t^2}+\frac{\partial Q_{13}}{\partial x}+\left(Q_{13}(P_3)+Q_{13}(q_3)\right)\right]\delta w\right\}dxdt+$$

$$+\int_t\left[\left(-N_x+N_x\left(\phi_1\right)\right)\delta u+\left(M_y-M_y\left(\phi_1\right)\right)\delta\alpha_1+\left(M_z-M_z\left(\phi_1\right)\right)\delta\alpha_2+\right.$$

$$\left.+\left(-Q_{12}+Q_{12}\left(\phi_2\right)\right)\delta v+\left(-M_x+M_x\left(\phi_2,\phi_3\right)\right)\delta\theta+\left(-Q_{13}+Q_{13}\left(\phi_3\right)\right)\delta w\right]\Big|_x dt+$$

$$+\int_x\left[\left(\rho F\frac{\partial u}{\partial t}-\rho S_y\frac{\partial\alpha_1}{\partial t}-\rho S_z\frac{\partial\alpha_2}{\partial t}\right)\delta u+\left(-\rho S_y\frac{\partial u}{\partial t}+\rho I_y\frac{\partial\alpha_1}{\partial t}+\rho I_{yz}\frac{\partial\alpha_2}{\partial t}\right)\delta\alpha_1+\right.$$

$$\begin{aligned}&+\left(-\rho S_z\frac{\partial u}{\partial t}+\rho I_{yz}\frac{\partial\alpha_1}{\partial t}+\rho I_z\frac{\partial\alpha_2}{\partial t}\right)\delta\alpha_2+\left(\rho F\frac{\partial v}{\partial t}+\rho S_y\frac{\partial\theta}{\partial t}\right)\delta v+\\&\left.+\left(\rho S_y\frac{\partial v}{\partial t}+\rho\left(I_y+I_z\right)\frac{\partial\theta}{\partial t}-\rho S_z\frac{\partial w}{\partial t}\right)\delta\theta+\left(\rho F\frac{\partial w}{\partial t}-\rho S_z\frac{\partial\theta}{\partial t}\right)\delta w\right]\Big|_t dx=0\end{aligned}\qquad(15)$$

Variational equation (15) consists of three main parts. The first part is an integral of the first kind. Here, the components of the displacement vector are unknown functions. Therefore, their variations are not equal to zero, which means the coefficients of variation of the function must be identically equal to zero.

The second part of the variational equation (15) describes the boundary conditions. In this part of the expression, the components of the displacement vector will not always be unknown functions. In geometric boundary conditions, the displacement components are specified. In this case, the variations of the known functions are zero, and under static boundary conditions, the expressions in parentheses are zero. Therefore, in the second part of the variational equation, each term with a variation factor of the displacement vector components is zero. These expressions characterize mixed boundary conditions; therefore, the expressions in parentheses are zero with the variation factors of the displacement vector components.

The third part of the variational equation (15) describes the initial conditions. The initial conditions specify the velocity and displacement.

Thus, from the variational equation (15) we obtain the following system of partial derivatives of differential equations with natural boundary and initial conditions:

$$-\rho F\frac{\partial^2 u}{\partial t^2}+\rho S_y\frac{\partial^2\alpha_1}{\partial t^2}+\rho S_z\frac{\partial^2\alpha_2}{\partial t^2}+\frac{\partial N_x}{\partial x}=-\left(N_x(P_1)+N_x(q_1)\right);$$

$$\rho S_y\frac{\partial^2 u}{\partial t^2}-\rho I_y\frac{\partial^2\alpha_1}{\partial t^2}-\rho I_{yz}\frac{\partial^2\alpha_2}{\partial t^2}-\frac{\partial M_y}{\partial x}+Q_{13}=\left(M_y(P_1)+M_y(q_1)\right);$$

$$\rho S_z\frac{\partial^2 u}{\partial t^2}-\rho I_{yz}\frac{\partial^2\alpha_1}{\partial t^2}-\rho I_z\frac{\partial^2\alpha_2}{\partial t^2}-\frac{\partial M_z}{\partial x}+Q_{12}=\left(M_z(P_1)+M_z(q_1)\right);$$

$$-\rho F\frac{\partial^2 v}{\partial t^2}-\rho S_y\frac{\partial^2\theta}{\partial t^2}+\frac{\partial Q_{12}}{\partial x}=-\left(Q_{12}(P_2)+Q_{12}(q_2)\right);$$

$$-\rho S_y\frac{\partial^2 v}{\partial t^2}-\rho\left(I_y+I_z\right)\frac{\partial^2\theta}{\partial t^2}+\rho S_z\frac{\partial^2 w}{\partial t^2}+\frac{\partial M_x}{\partial x}=-\left(M_x(P_2,P_3)+M_x(q_2,q_3)\right);$$

$$-\rho F\frac{\partial^2 w}{\partial t^2}+\rho S_z\frac{\partial^2\theta}{\partial t^2}+\frac{\partial Q_{13}}{\partial x}=-\left(Q_{13}(P_3)+Q_{13}(q_3)\right). \quad (16)$$

boundary conditions

$$\left(\left(-N_x+N_x\left(\phi_1\right)\right)\right)\delta u\Big|_x=0;$$

$$\left(M_y-M_y\left(\phi_1\right)\right)\delta\alpha_1\Big|_x=0;$$

$$\left(M_z-M_z\left(\phi_1\right)\right)\delta\alpha_2\Big|_x=0;$$

$$\left(-Q_{12}+Q_{12}\left(\phi_2\right)\right)\delta\overline{v}\Big|_x=0;$$

$$\left(-M_x+M_x\left(\phi_2,\phi_3\right)\right)\delta\theta\Big|_x=0;$$

$$\left(-Q_{13}+Q_{13}\left(\phi_3\right)\right)\delta w\Big|_x=0. \quad (17)$$

initial conditions

$$\left(\rho F\frac{\partial u}{\partial t}-\rho S_y\frac{\partial\alpha_1}{\partial t}-\rho S_z\frac{\partial\alpha_2}{\partial t}\right)\delta u\Bigg|_t=0;\ \left(-\rho S_y\frac{\partial u}{\partial t}+\rho I_y\frac{\partial\alpha_1}{\partial t}+\rho I_{yz}\frac{\partial\alpha_2}{\partial t}\right)\delta\alpha_1\Bigg|_t=0;$$

$$\left(-\rho S_z\frac{\partial u}{\partial t}+\rho I_{yz}\frac{\partial\alpha_1}{\partial t}+\rho I_z\frac{\partial\alpha_2}{\partial t}\right)\delta\alpha_2\Bigg|_t=0;$$

$$\left(\rho F\frac{\partial v}{\partial t}+\rho S_y\frac{\partial\theta}{\partial t}\right)\delta v\Bigg|_t=0;$$

$$\left(\rho S_y\frac{\partial v}{\partial t}+\rho\left(I_y+I_z\right)\frac{\partial\theta}{\partial t}-\rho S_z\frac{\partial w}{\partial t}\right)\delta\theta\Bigg|_t=0;$$

$$\left(\rho F\frac{\partial w}{\partial t}-\rho S_z\frac{\partial \theta}{\partial t}\right)\delta w\bigg|_t=0. \qquad (18)$$

where

ρ, E and G - mechanical parameters of rod systems;

F, I_y, S_y, I_z, S_z and I_{yz} - geometric parameters of rod systems;

$N_x(P_1)$, $Q_{12}(P_2)$ and $Q_{13}(P_3)$ – volumetric longitudinal and transverse forces;

$M_y(P_1)$, $M_z(P_1)$ and $M_x(P_2,P_3)$ – volumetric bending and torsional moments;

$N_x(q_1)$, $Q_{12}(q_2)$ and $Q_{13}(q_3)$ – surface longitudinal and transverse forces;

$M_y(q_1)$, $M_z(q_1)$ and $M_x(q_2,q_3)$ - surface bending and torsional moments;

$N_x(\phi_1)$, $Q_{12}(\phi_2)$ and $Q_{13}(\phi_3)$ – end longitudinal and transverse forces;

$M_y(\phi_1)$, $M_z(\phi_1)$ and $M_x(\phi_2,\phi_3)$ - end bending and torsional moments.

Conclusion

This article presents a new mathematical framework for calculating beam systems under spatially dynamic loading in a rectangular coordinate system.

The system of partial differential equations (16), boundary conditions (17), and initial conditions (18) constitutes a general mathematical model for the dynamic analysis of beam systems under complex spatial loading.

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