ISSN 2181-8622

Manufacturing technology problems

Scientific and Technical Journal Namangan Institute of Engineering and Technology

Volume 10
Issue 3
2025

TRANSITION FUNCTION OF SECOND-ORDER ELEMENT

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Abstract: This work analyzes the denominator of the transfer function of second-order elements widely used in modern automatic control systems - the characteristic equation. The transfer function is calculated and derived, and it is proposed to use the expression of this derived transfer function as a mathematical model for the convenience of identification of second-order inertial elements based on experimental transition characteristics.

Keywords: technological process, transfer function, transition function, characteristic equation, time constant, identification, mathematical model, model parameters.

Introduction. In automated control systems for technological processes, production, and their complexes, it is necessary to determine the behavior of the control object, the course of processes occurring in that object, and the patterns in them, as well as to introduce corrections. According to the analysis performed in [1], as a result of such determination and introduction of corrections, the necessity arises to form the influence on the object that ensures good results, that is, when creating control systems for technological processes or industrial objects, the necessity arises to have mathematical descriptions of the control object, control device, and other elements included in this system. Finding the above-mentioned influence belongs to the problems of building a control model based on "input-output" data, i.e., identification problems, and its relevance is clearly visible in the automation of technological processes, production, and their complexes.

Obtaining experimental data related to "input-output" while the object is operating has certain difficulties. In such cases, reference is made to its mathematical model, which allows preliminary study of the object. Mathematical models of objects are differential equations in many cases [2, 3]. In this regard, according to the order of differential equations and the numerical values of parameters in them, objects are divided into 1st-order, 2nd-order inertial, oscillatory, conservative, and other objects [3, 4].

Problem statement. When evaluating the parameters of transition characteristics based on experimental data related to "input-output" of objects, a mathematical model with parameters of these characteristics is selected in advance. Here, it is necessary to select a mathematical model or write it in such a form that finding the parameters of this model from experimental data is convenient.

In practice, identification of the transfer function of first-order inertial elements is more common, which is well covered in [4, 5]. However, in this case, approximation has rough errors. We hope that approximation gives good results for 2nd-order inertial elements ($\xi = \dfrac{T_2}{2T_1} > 1$). Here also certain difficulties exist. The transfer function of a second-order non-oscillatory element is given as follows:

$$h(t) = k + C_1 e^{-s_1 t} + C_2 e^{-s_2 t}$$

The identification of the transfer function of a given element is presented in [5]. This is indeed true, but there is complexity in determining its parameters k, C_1, C_2, s_1, s_2, for example, this complexity lies in the large number of parameters. In [1, 6], the significant aspects of these parameters for the transition process are shown. If we could write the above transfer function in such a way that the number of parameters decreases, then identification would also be simplified. The problem of this work consists of this.

Methodology and empirical analysis. To solve the problem posed in this work, the mathematical model of the second-order object, the calculated transfer function, characteristic equation, and the transition process in it are analyzed.

The transfer function of this element is:

$$W(p) = \frac{k}{T_1^2 p^2 + T_2 p + 1}. \tag{1}$$

The characteristic equation is:

$$T_1^2 p^2 + T_2 p + 1 = 0. \tag{2}$$

This is a simple quadratic equation. Its roots are found using the following formula:

$$p_{1,2} = \frac{-T_2 \pm \sqrt{T_2^2 - 4T_1^2}}{2T_1^2} \tag{3}$$

Obviously, depending on the magnitude of the expression under the root, the roots will be either real or complex numbers

$$\sqrt{T_2^2 - 4T_1^2}.$$

If the following relationship holds:

$$T_2^2 - 4T_1^2 \geq 0,$$

we can write the following:

$$T_2^2 \geq 4T_1^2. \tag{4}$$

For the damping coefficient:

$$\xi = \frac{T_2}{2T_1} > 1.$$

The 2nd-order element in this case is called an inertial (non-oscillatory) 2nd-order element.

If the following inequality holds for the expression under the root in (3):

$$T_2^2 - 4T_1^2 < 0,$$

the following relationship is obtained for time constants:

$$T_2^2 < 4T_1^2, \tag{5}$$

in this case, the damping coefficient:

$$\xi = \frac{T_2}{2T_1} < 1.$$

The 2nd-order element in this case becomes an oscillatory element.

An ideal oscillatory link is one in which undamped oscillations occur: for it the damping coefficient ξ=0.

Obviously, this is possible only if the time constant $T_2 = 0$.

In this case, the oscillatory link becomes a conservative link.

If $T_1 = 0$ is taken for a 2nd-order link, the damping coefficient takes the following value

$$\xi = \infty.$$

This means that the oscillations cannot be fundamental. In this case, a 1st-order aperiodic link is obtained.

The transition processes of a 2nd-order link are calculated according to the representation of its transition function:

$$H(p) = \frac{k}{p(T_1^2 p^2 + T_2 p + 1)}. \tag{6}$$

To find its original, that is, the transition function, it is necessary to factor the expression in parentheses in the denominator of (6). To do this, it is necessary to determine the roots of the characteristic equation (2), which is necessary to determine the relationship between the time constants (4) and (5).

The roots of the characteristic equation for this term are calculated as follows

$$p_1 = \frac{-T_2 \pm \sqrt{T_2^2 - 4T_1^2}}{2T_1^2} = -\alpha_1;\; p_2 = \frac{-T_2 \pm \sqrt{T_2^2 - 4T_1^2}}{2T_1^2} = -\alpha_2.$$

Obviously, they are real and negative.

According to the factorization rule:

$$T_1^2 p^2 + T_2 p + 1 = T_1^2 (p + \alpha_1)(p + \alpha_2).$$

The image of the transition function can now be written as

$$H(p) = \frac{k}{T_1^2} \frac{1}{p(p + \alpha_1)(p + \alpha_2)}.$$

The following original corresponds to the above image:

$$h(t) = \frac{k}{T_1^2} \frac{1}{\alpha_1 \alpha_2 (\alpha_1 - \alpha_2)} [(\alpha_1 - \alpha_2) + \alpha_2 e^{-\alpha_1 t} - \alpha_1 e^{-\alpha_2 t}]. \tag{7}$$

Let's simplify this expression. To do this, we find the product $\alpha_1 \alpha_2$ separately:

$$\alpha_1 \alpha_2 = \frac{T_2 + \sqrt{T_2^2 - 4T_1^2}}{2T_1^2} * \frac{T_2 - \sqrt{T_2^2 - 4T_1^2}}{2T_1^2} = \frac{T_2^2 - (T_2^2 - 4T_1^2)}{2T_1^4} = \frac{1}{T_1^2} \tag{8}$$

Therefore,

$$\frac{k}{T_1^2} \frac{1}{\alpha_1 \alpha_2} = \frac{k}{T_1^2} T_1^2 = k.$$

In this case, the transition function takes the following form

$$
\begin{aligned}
h(t) = \frac{k}{(\alpha_1-\alpha_2)}\left[(\alpha_1-\alpha_2)+\alpha_2 e^{-\alpha_1 t}-\alpha_1 e^{-\alpha_2 t}\right] = \\
= k\left[\frac{(\alpha_1-\alpha_2)}{(\alpha_1-\alpha_2)}+\frac{\alpha_2 e^{-\alpha_1 t}}{(\alpha_1-\alpha_2)}-\frac{-\alpha_1 e^{-\alpha_2 t}}{(\alpha_1-\alpha_2)}\right] \\
= k\left[1-\frac{\alpha_2 e^{-\alpha_1 t}}{(\alpha_2-\alpha_1)}\right. \\
\left.+\frac{\alpha_1 e^{-\alpha_2 t}}{(\alpha_2-\alpha_1)}\right]. \qquad (9)
\end{aligned}
$$

It is clear that expression (9) consists of the sum of two exponentials. Therefore, the second-order link is also called the second-order inertia link, assuming the relation $T_2 \geq 2T_1$ for the above-mentioned time constants.

This conclusion is also supported by the fact that the transfer function of this link can also be represented in the following form:

$$
W(p) = \frac{k}{T_1^2}\frac{1}{(p+\alpha_1)(p+\alpha_2)} = \frac{k}{\alpha_1\alpha_2}\cdot\frac{1}{\frac{1}{\alpha_1}p+1}\cdot\frac{1}{\frac{1}{\alpha_2}p+1}.
$$

Finally, we can write the following

$$
W(p) = \frac{k_1}{\frac{1}{\alpha_1}p+1}\cdot\frac{k_2}{\frac{1}{\alpha_2}p+1}. \qquad (10)
$$

This expression corresponds to the series connection of two inertial links, the time constants of which are written below:

$$
T_1 = \frac{1}{\alpha_1}, \qquad T_2 = \frac{1}{\alpha_2}, \qquad (11)
$$

and the amplification coefficients are k_1 and k_2, respectively, the magnitudes of these coefficients are determined from $k_1 k_1 = k$.

Expression (9) for the transition function can be written as follows, taking into account (11):

$$
\begin{aligned}
h(t) = \left[\left(k-\frac{k\alpha_2 e^{-\alpha_1 t}}{(\alpha_2-\alpha_1)}\right)+\left(\frac{k\alpha_1 e^{-\alpha_2 t}}{(\alpha_2-\alpha_1)}\right)\right] = \\
= k\left[\left(1-\frac{\frac{1}{T_2}e^{-\frac{t}{T_1}}}{\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}\right)+\left(\frac{\frac{1}{T_1}e^{-\frac{t}{T_2}}}{\frac{1}{T_2}-\frac{1}{T_1}}\right)\right] = k\left[\left(1-\frac{\frac{T_1T_2}{T_2}e^{-\frac{t}{T_1}}}{\left(\frac{T_1T_2}{T_2}-\frac{T_1T_2}{T_1}\right)}\right)+\left(\frac{\frac{T_1T_2}{T_1}e^{-\frac{t}{T_2}}}{\frac{T_1T_2}{T_2}-\frac{T_1T_2}{T_1}}\right)\right] = \\
= k\left[\left(1-\frac{T_1 e^{-\frac{t}{T_1}}}{T_1-T_2}\right)+\left(\frac{T_2 e^{-\frac{t}{T_2}}}{T_1-T_2}\right)\right] = \\
= k\left(1-\frac{T_1}{T_1-T_2}e^{-\frac{t}{T_1}}+\frac{T_2}{T_1-T_2}e^{-\frac{t}{T_2}}\right).
\end{aligned}
$$

Results. In the transition function of the pre-existing second-order link,

$$
h(t) = k + C_1 e^{-s_1 t} + C_2 e^{-s_2 t}
$$

k, C_1, C_2, s_1, s_2 are five parameters, which are difficult to identify using the experimental transition characteristic.

The following expression was calculated for the transition function of the second-order link:

$$h(\mathrm{t}) = k\left(1 - \frac{T_1}{T_1 - T_2} e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} e^{-\frac{t}{T_2}}\right)$$

The resulting expression contains three parameters k, T_1, T_2 This makes it much easier todetermine these parameters from the experimentally given transition characteristic.

Conclusion. In identifying the second-order inertial link $(\xi = \frac{T_2}{2T_1} > 1)$ by the experimental transition function, it is proposed to take expression (8) as a mathematical model for the transition function. The identification of the parameters k, T_1, T_2 in this model is relatively easy.

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