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THEORETICAL STUDY OF BENDING VIBRATIONS OF PACKET-TYPE WORKING BODIES OF TECHNOLOGICAL MACHINES WITH ACCOUNT OF INTERNAL LONGITUDINAL FORCES

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Abstract: This paper presents a theoretical investigation of the dynamics of packet-type working bodies of technological machines under bending vibrations. Structural configurations with relatively short packets of disk elements and short shafts are considered. Expressions for the potential and kinetic energies of the system are obtained with allowance for shear deformations and rotary inertia of cross-sections. Based on Hamilton's principle, the differential equations of motion are derived. An analysis of the influence of internal longitudinal forces forming a closed force contour is performed. It is established that the variation of the system potential energy is determined by the ratio of longitudinal stiffnesses of the elements.

Keywords: machine dynamics, bending vibrations, packet-type working body, vibrations, stiffness, energy method.

Introduction. Modern technological machines are characterized by high speeds of working processes, which lead to an increase in dynamic loads acting on their working bodies. One of the widely used structural configurations is the packet-type working body consisting of a set of disk elements mounted on a shaft [1].

During the operation of such systems, the following phenomena are observed:

- bending vibrations;
- longitudinal force interactions between elements;
- variation of the energy state of the structure.

Particularly important is the analysis of systems with relatively short packets of disk elements, represented in the calculation schemes (Figs. 1 and 2).

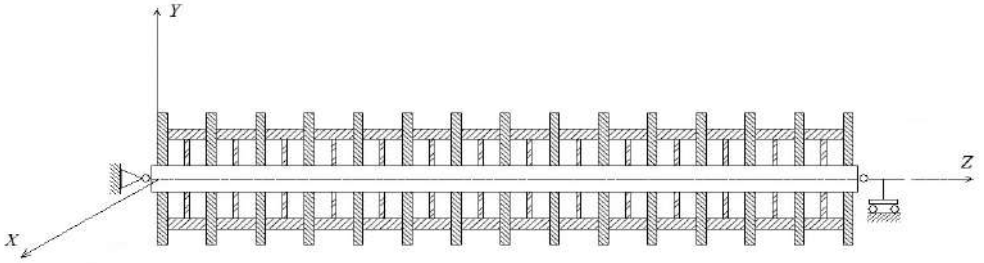

Fig. 1. Calculation scheme of a packet-type working body with a relatively short packet of disk elements

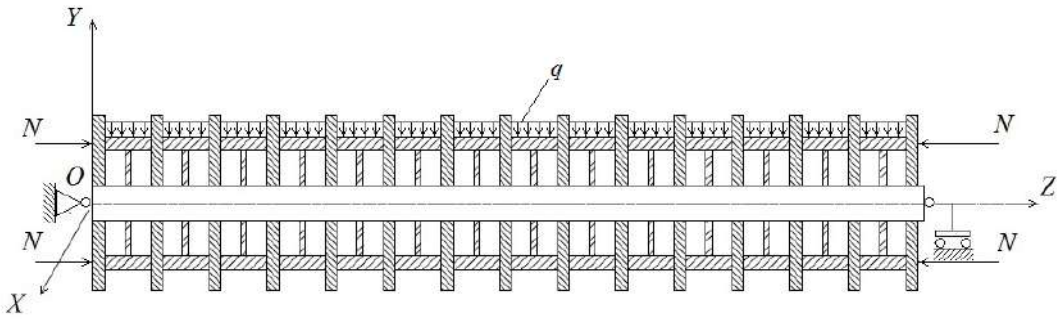

Fig. 2. Scheme of longitudinal force action in a packet-type working body under bending vibrations

Under these conditions, the classical beam theory proves to be insufficient, since it becomes necessary to take into account:

- shear deformations;
- rotation of cross-sections;
- internal force factors.

Theoretical Foundations of Modeling

Potential Energy

Consider a packet-type working body of length LLL, performing bending vibrations in the vertical plane.

The potential energy of the system is determined as the sum of:

- bending energy;
- shear energy;
- energy of longitudinal deformations.

The general expression has the form:

$$U = \int_0^L \left(EJ \left(\frac{\partial^2 u}{\partial z^2} \right)^2 + \chi GF \left(\frac{\partial u}{\partial z} - \beta \right)^2 \right) dz \qquad (1)$$

Kinetic Energy

The kinetic energy includes:

- translational motion of disk elements;
- rotational motion of cross-sections;
- inertial effects of the packet.

$$- T = \frac{1}{2} \int_0^L \left(\rho F \dot{u}^2 + \rho J \dot{\beta}^2 \right) dz \qquad (2)$$

Derivation of the Differential Equation of Motion

Applying Hamilton's principle

$$\delta = \int_{t_1}^{t_2} (T - U) dt = 0$$

a system of equations describing bending vibrations is obtained:

$$EJ \frac{\partial^4 u}{\partial z^4} + \rho F \frac{\partial^2 u}{\partial t^2} + \chi GF \frac{\partial^2 u}{\partial z^2} = 0$$

(3)

The equation reflects the influence of:
- bending stiffness;
- shear stresses;
- distributed mass.

Vibrations in the Case of a Short Shaft

For a structure with a short shaft (Fig. 3), the energy model becomes more complex.

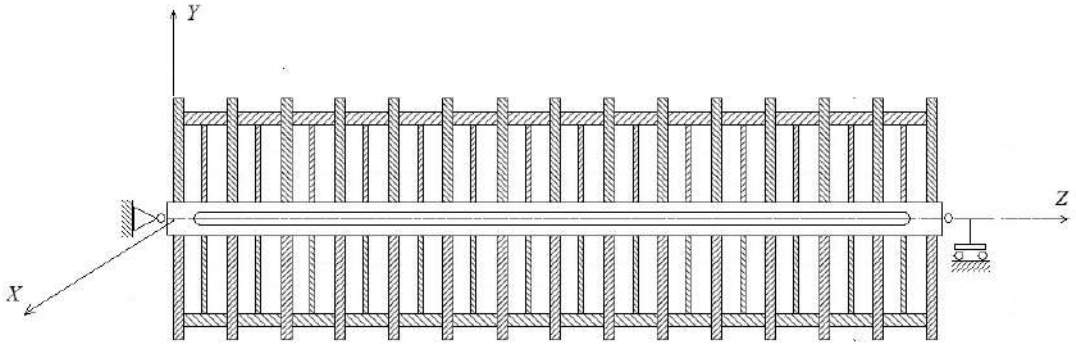

Fig. 3. Calculation scheme of a packet-type working body with a relatively short shaft and a packet of disk elements

The potential energy of the system takes the form:

$$U = U_b + U_n \tag{4}$$

where deformations of both the shaft and the packet of disk elements are taken into account[2].

The corresponding equation of motion is:

$$EJ\frac{\partial^4 u}{\partial z^4} + k_s\frac{\partial^2 u}{\partial z^2} + \rho\frac{\partial^2 u}{\partial t^2} = 0 \tag{5}$$

Influence of Internal Longitudinal Forces

A distinctive feature of packet-type working bodies is the presence of an internal closed force contour arising as a result of [3,4]:
- tension of the shaft;
- compression of the packet of disk elements.

This is illustrated in the calculation scheme (Fig. 4).

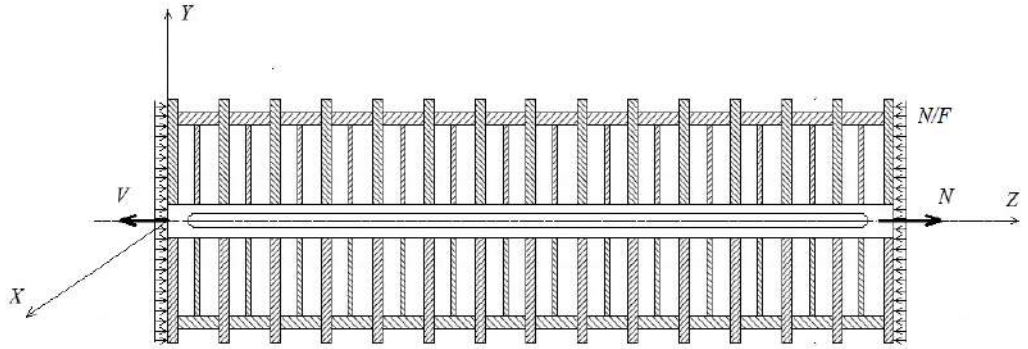

Fig. 4. Calculation model of a packet-type working body with an internal closed force contour

The potential energies of the elements are given by:

$$U_{Nb} = \frac{1}{2}\int_0^L \frac{N^2}{E_b F_b} dz \tag{6}$$

$$U_{Nn} = -\frac{1}{2}\int_0^L \mu N^2 dz \tag{7}$$

During bending vibrations, an additional longitudinal deformation occurs:

$$\varepsilon' \approx \frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2 \tag{8}$$

The total variation of the potential energy is:

$$\Delta U = \int_0^L (E_b F_b - E_n F_n)\varepsilon'^2 dz \tag{9}$$

The analysis shows that:

- when the longitudinal stiffnesses are equal

$$\Delta U = 0$$

- when they differ, the natural frequencies undergo variation.

Discussion

The obtained results make it possible to conclude that:

- internal force factors can alter the dynamic characteristics of the system;
- their influence is determined by the ratio of stiffnesses;
- an increase in bending stiffness enhances the structural stability;
- an increase in mass leads to a decrease in natural frequencies.

The practical significance of the results lies in the possibility of:

- calculating critical operating regimes of machines;
- optimizing the parameters of working bodies;
- increasing the service life of equipment.

Conclusion

1. A mathematical model of bending vibrations of packet-type working bodies has been developed.
2. The influence of internal longitudinal forces on the energy state of the system has been established.
3. It has been shown that the variation of potential energy is determined by the ratio of longitudinal stiffnesses.
4. The obtained relationships can be used in the design of high-speed technological machines.

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