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BENDING VIBRATIONS OF FLEXIBLE PACKET-TYPE WORKING BODIES OF TECHNOLOGICAL MACHINES

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Abstract: This paper examines the patterns governing the initiation and development of bending vibrations in flexible packet-type working bodies of technological machines. The principal types of transverse forces forming the dynamic state of the structure are analyzed. Based on Hamilton's variational principle, differential equations describing free bending vibrations are derived. A generalized representation of the mass distribution of disk elements is developed, and boundary conditions are formulated for various methods of fixing the working body. The obtained relationships can be applied in the calculation of critical rotational speeds, vibration stability, and dynamic reliability of machines.

Keywords: flexible packet-type working body, bending vibrations, critical frequency, machine dynamics, elastic axis, damping.

Introduction. Working bodies of technological machines designed in the form of packet-type structures are widely used in equipment for the textile, processing, and other industrial sectors. During operation, these elements are subjected to complex dynamic loads caused by the non-uniformity of technological processes, random external influences, and structural features.

Under the action of transverse forces, bending vibrations arise in such structures, significantly affecting the reliability and performance quality of machines. Of particular concern are operating regimes close to resonance conditions, when the frequency of forced vibrations coincides with or approaches the natural frequency of the structure.

In this regard, the investigation of bending vibrations of flexible packet-type working bodies represents an important scientific and engineering task aimed at improving the efficiency of technological equipment design.

1. Sources of Bending Vibrations

During operation, packet-type working bodies are subjected to the action of transverse forces of various origins (Fig. 1). These forces may be classified as follows:

Forces of static and dynamic unbalance, caused by the eccentricity of mass distribution relative to the axis of rotation.

Forces of technological resistance of the processed material, which vary with time and along the length of the working body.

Forces of random external disturbances, whose magnitude and periodicity are determined by operating conditions.

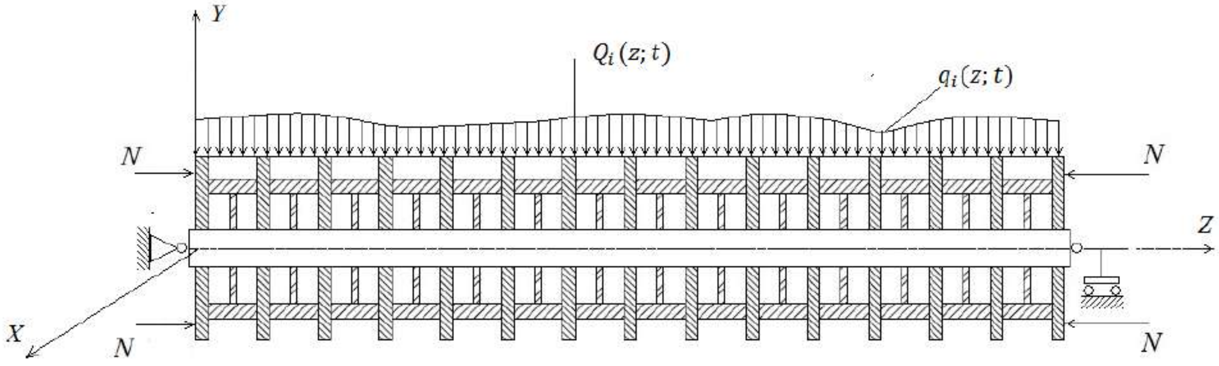

Fig. 1. Diagram of the action of transverse forces on a packet-type working body

Under the influence of the above-mentioned factors, both free and forced bending vibrations arise. Free vibrations decay rapidly due to the significant structural damping inherent in packet-type systems. Forced vibrations, in contrast, may be sustained by external excitations and can lead to the development of dangerous amplitudes when the excitation frequencies coincide with the natural frequencies of the system.

2. Calculation Model of a Flexible Packet-Type Working Body

The calculation scheme of a flexible packet-type working body is shown in Fig. 2.

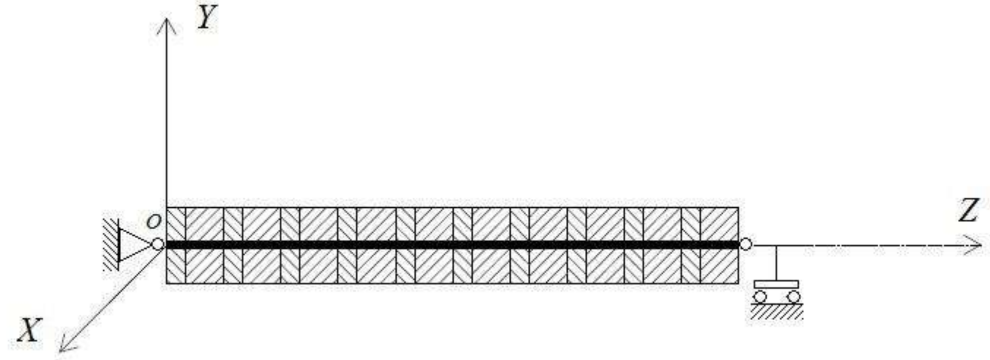

Fig. 2. Calculation model of a flexible packet-type working body

The following assumptions are adopted:

- the axis of the working body is straight and coincides with the axis xxx;
- the centers of gravity of the cross-sections of the disk elements are located on this axis;
- bending vibrations occur in the plane xOyxOyxOy;
- cross-sections remain plane during deformation and perpendicular to the deformed axis;
- axial extension during bending is neglected.

Under these conditions, the central axis of the structure coincides with the centroid of the cross-sections and is considered as the elastic axis of the working body.

3. Generalized Mass Distribution

The mass of the disk elements is distributed discretely along the length of the working body. However, due to the small thickness of the disks compared to the length of the structure (by 2–3 orders of magnitude), it is permissible to transition to a continuous model.

The mass distribution functions of the working and spacer disks are defined by the expressions

$$f_{mpi} = \rho_p F_{pi}(z), \qquad f_{mni} = \rho_n F_{ni}(z)$$

where: ρ_p, ρ_n - material densities;

$F_{pi}(z), F_{ni}(z)$ - functions of the cross-sectional areas.

Taking into account the uniform alternation of disks, a generalized mass distribution function is introduced:

$$f_n(z) = \frac{\rho_p F_p l_p + \rho_n F_n l_n}{l_p + l_n}$$

Such an approach makes it possible to transition from a discrete dynamic system to a continuous beam model with distributed parameters.

4. Energy Formulation of the Problem

To derive the equations of bending vibrations, Hamilton's variational principle is employed.

The potential energy of deformation is expressed as

$$U = \frac{1}{2}\int_0^L D(z)\left(\frac{\partial^2 u}{\partial z^2}\right)^2 dz \tag{1}$$

where: $D(z)$ - equivalent bending stiffness of the packet-type working body.

The kinetic energy is defined by the expression

$$T = \frac{1}{2}\int_0^L f_n\left(\frac{\partial u}{\partial t}\right)^2 dz \tag{2}$$

where: $u(z,t)$- transverse deflection of the points along the axis of the working body.

Application of the Lagrange equations of the second kind leads to the differential equation of free bending vibrations:

$$D(z) = \frac{d^4 u}{dz^4} + f_n(z)\frac{d^2 u}{dt^2} = 0 \tag{3}$$

The obtained equation describes the dynamic state of a flexible packet-type working body taking into account distributed parameters.

5. Initial and Boundary Conditions

The solution of equation (3) is possible only after specifying the initial and boundary conditions.

The initial conditions are defined by the functions

$$u(z,0) = f(z), \qquad \frac{du}{dz}(z,0) = g(z)$$

The boundary conditions depend on the type of end support of the working body. The most common variants are shown in Figs. 3– 9:

1. Clamped Support

Zero displacements and zero rotation angles.

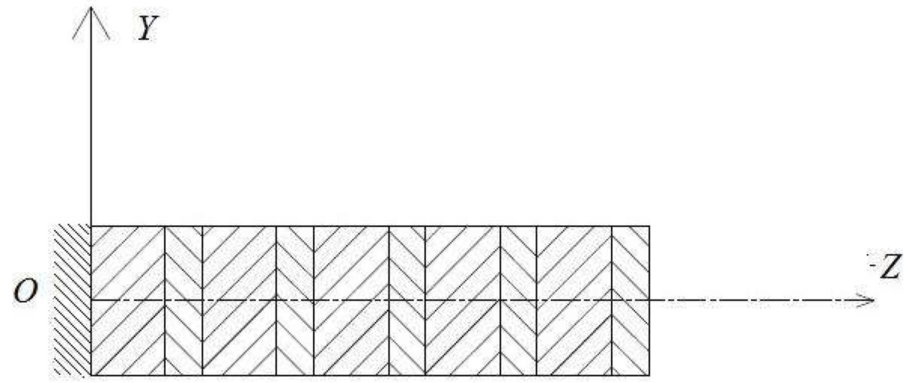

Fig. 3. Clamped support of a flexible packet-type working body

2. Simply Supported-Zero displacements and zero bending moment at the ends.

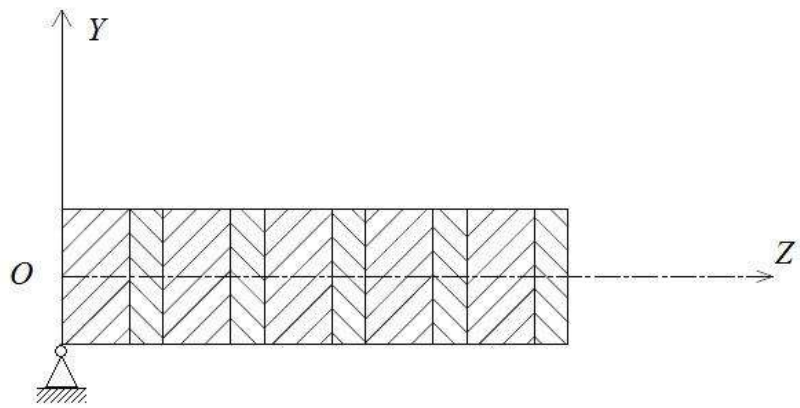

Fig. 4. Simply supported working body

3. Free End -Zero transverse force and zero bending moment at the end.

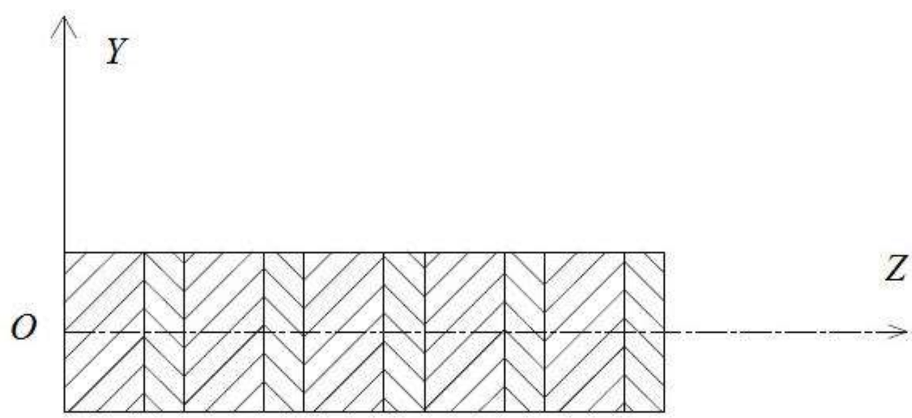

Fig. 5. Diagram of a working body with a free end

4. Floating Support -Zero transverse force and zero rotation angle.

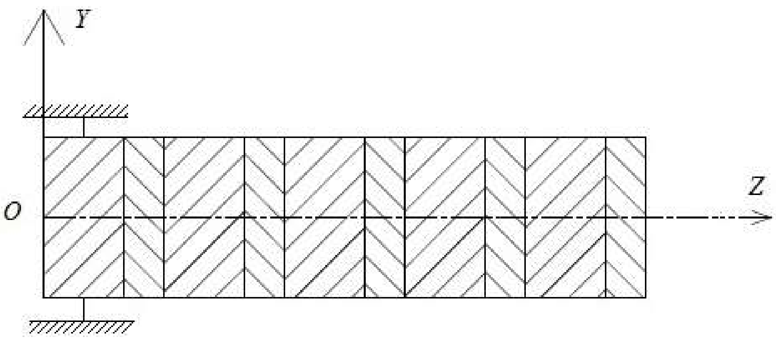

Fig. 6. Floating support of a working body

5. Vertically Elastic Support - The bending moment is zero, while the transverse force is determined by the elastic element.

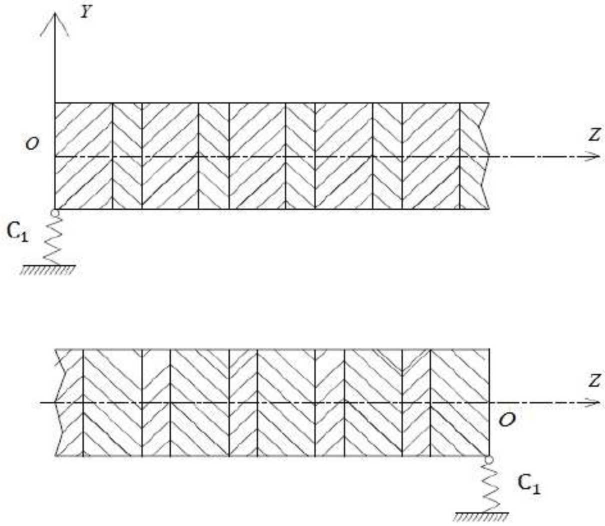

Fig. 7. Vertically elastic support of a working body

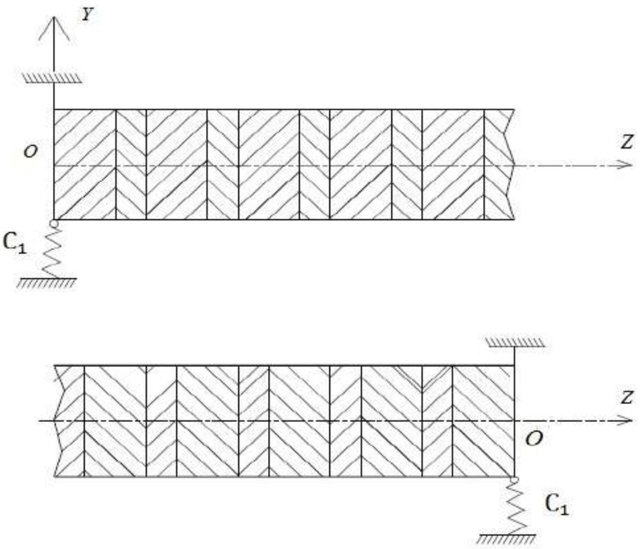

Fig. 8. Vertically elastic support in a floating configuration

6. Rotationally Elastic Support - The moment is equal to the reactive moment of the elastic element.

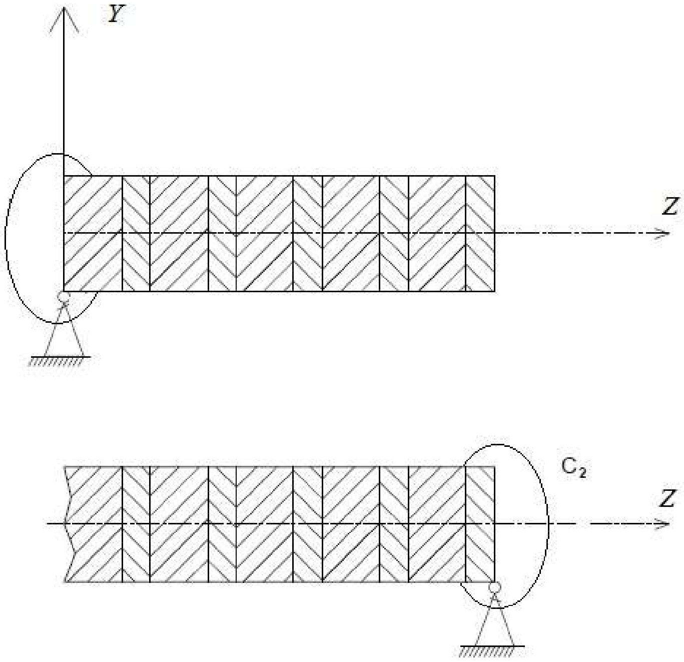

Fig. 9. Rotationally elastic support of a working body

Accounting for realistic support conditions significantly affects the natural frequencies and vibration mode shapes of the structure.

6. Analysis of Dynamic Characteristics

The **first critical rotational speed** of a packet-type working body has the greatest practical significance.

During machine operation, the following regimes may occur:

- **Underdamped regime**, characterized by rapid decay of free vibrations;
- **Forced vibrations**, sustained by external excitations;
- **Resonance regime**, where the vibration amplitude increases sharply.

A high bending stiffness of the packet-type structure and significant internal damping contribute to reducing vibration activity. However, increasing the length of the working body or reducing the initial pre-compression may lead to a transition to **dangerous operating regimes**.

Conclusions

1. It has been established that the bending vibrations of flexible packet-type working bodies are caused by a combination of **technological, structural, and random factors**.
2. A **continuous calculation model** has been developed, taking into account the **mass distribution** and **equivalent bending stiffness** of the packet-type structure.
3. Using the **energy method**, the differential equation of free bending vibrations has been derived.

4. It has been shown that the **support conditions** exert a determining influence on the **natural frequencies** and **vibration mode shapes**.

5. The obtained relationships can be used for the **calculation of critical operating regimes**, as well as for the **design and optimization of technological machines**.

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