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DESCRIBING COMPUTATIONAL DOMAINS IN APPLICATIONS FOR SOLVING THREE-DIMENSIONAL PROBLEMS OF TECHNOLOGICAL PROCESSES

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Abstract: This article discusses the problems encountered in the creation of computer programs for solving three-dimensional problems of mathematical physics. The complexity of describing areas of problem solving for a computer is explained. A method of representing three-dimensional areas in the form of combinations of geometric shapes is proposed. A method for obtaining images of a flat section of three-dimensional areas using the construction of a grid is discussed. A method for determining the location of points relative to three-dimensional areas is proposed.

Keywords: process control problem, application program, interior of the area, exterior of the area, boundary of the area, description of three-dimensional shapes, localization of points, flat section.

Introduction. The research of many physical processes in nature leads to the solution of various equations in abstract limited domains with some boundary conditions [1]. Solving such problems by analytical methods is very difficult or it is not possible. In such cases, numerical (approximate) methods of computational mathematics are applied to the solution. The essence of approximate methods is that first the problem is solved in a discrete image of a continuous domain. Then the convergence of a discrete solution to the analytical solution is proved with an infinite increase in the number of points that make up the discrete domain.

The discretization of the domain can be carried out in many ways. The most famous of them are the constructions of a finite difference grid [2], a finite element grid [3], a finite volume grid [4] and a grid consisting of a stochastic set of points [5].

1. The structure of application programs.

With the computer implementation of such approximate methods for describing the computational domain and determining the location (localization) of grid nodes relative to it is a voluminous work in terms of the complexity of the methods and the number of required operations.

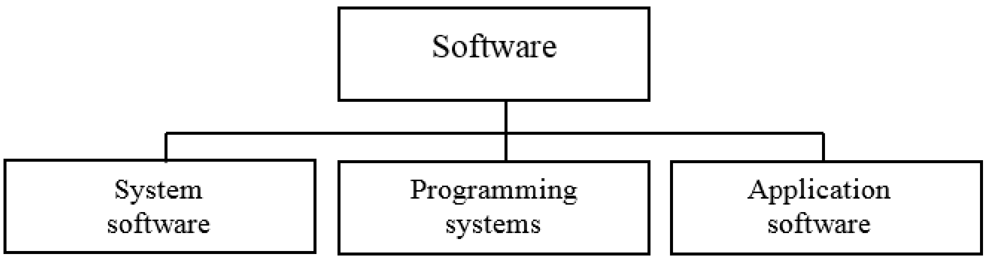


Figure 1. Types of software

This article suggests one of the ways to computer describe three-dimensional regions with a piecewise smooth boundary.

Software is a set of all programs and relevant documentation that ensures the use of a computer in the interests of each of its users. In a simplified form, the software can be represented as the following diagram (Fig.1).

Application software is a set of programs for solving problems of a certain class of a specific subject area. Usually, application programs are called applications.

They include:

- word processors;
- table processors;
- databases;
- integrated packages;
- illustrative and business graphics systems (Graphic processors);
- expert systems;
- training programs;
- programs for mathematical calculations, modeling and analysis;
- games;
- communication programs.

Technological processes are complex physical phenomena. To control them, it is necessary to create a mathematical model of the process, the result of it is a mathematical physics problem. Consequently, among the listed types of software, we are interested in "programs for mathematical calculations, modeling and analysis".

Creating application programs to solve such problems is quite a difficult and time-consuming job. First of all, the difficulty lies in translating the physical (or geometric) domain of the problem into the language of numbers that a computer can operate with. To understand the importance of this task, let's look at the following scheme for developing an application program (Fig.2).

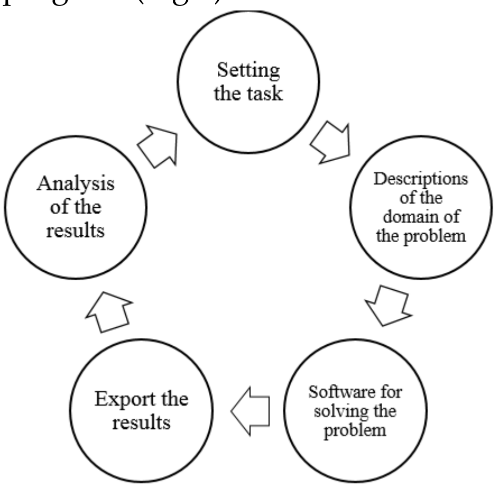


Figure 2. Scheme of application software development

It can be seen on this diagram that without a digital description of the computational domain of the problem being solved, we cannot proceed to creating a program for solving it. Below we will introduce the concept of the input language and propose a way to describe some three-dimensional areas.

2. The concept of an input language for describing the domain of problem solving.

In the process of solving three-dimensional problems, a digital description of an arbitrary spatial area where the task is to be solved is difficult and time-consuming one. Currently, there is no universal way to describe the computational domains of boundary value problems. Therefore, each application program for solving problems should have its own way of describing the geometric domain. Such methods of description are called input languages. In addition, among the existing methods, there are no ways to describe three-dimensional shapes with an absolutely non-smooth boundary: the boundary of the computational domain must be at least piecewise smooth one.

As an example of an input language, let's consider one of the ways to describe three-dimensional areas. One of the ways to digitally describe three-dimensional regions with piecewise smooth boundaries is to represent them as intersections of half-spaces bounded by first- and second-order surfaces. In this case, the boundary of the computational domain is approximated using pieces of smooth surfaces of the second order and planes. Each section of the figure that is limited by some part of the plane or surface of the second order corresponds to one half-space bounded by this surface. Each surface is defined by the coefficients of the general equation. Each plane corresponds to the coefficients A, B, C, D of the equation

$$Ax + By + Cz + D= 0.$$

And the second order surfaces correspond to the coefficients A, B, C, D, E, F, G, H, P, Q of the equation

$$Ax^2 + By^2 + Cx^2 + Dxy + Exz + Fyz + Gx + Hy + Pz + Q = 0.$$

In addition, each surface has its own unique number. Then we have the opportunity to describe a three-dimensional figure using the intersection and negation operations. Let's explain this description with an example. Let's assume that our figure has the form of a cube with a face, for example, 10 cm long and with a spherical void inside, with a radius of 2 cm. A schematic representation of this three-dimensional area is shown in Fig.3.

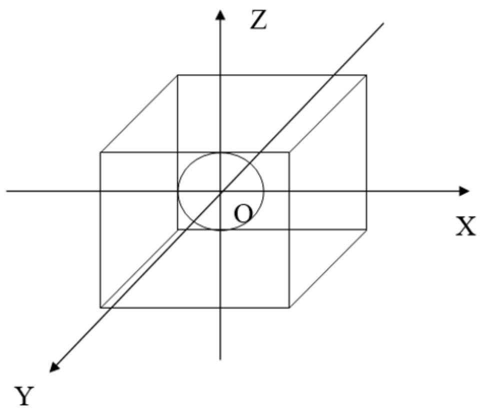


Figure 3. A schematic representation of our three-dimensional area

Surface 1: $x - 5 = 0$. Corresponding to it coefficients are: 1; 0; 0; -5.
Surface 2: $x + 5 = 0$. Corresponding to it coefficients are: 1; 0; 0; 5.
Surface 3: $y - 5 = 0$. Corresponding to it coefficients are: 0; 1; 0, -5.
Surface 4: $y+- 5 = 0$. Corresponding to it coefficients are: 0; 1; 0, 5.
Surface 5: $z - 5 = 0$. Corresponding to it coefficients are: 0; 0; 1; -5.
Surface 6: $z+ 5 = 0$. Corresponding to it coefficients are: 0; 0; 1; 5.
Surface 7: $x^2 + y^2 + z^2 - 4 = 0$. Corresponding to it coefficients are: 1; 1; 1; 0; 0; 0; 0; 0; 0; -4.

Now the specified area can be described using surface numbers in the form $1 \cap \overline{2} \cap 3 \cap \overline{4} \cap 5 \cap \overline{6} \cap \overline{7}$ which is an array (1,-2,3,-4,5,-6,-7) for a computer.

With this representation of regions, it is possible to localize (check the affiliation) arbitrary points relative to a given area. Below is one of the methods of localization of points, developed by A.Turakulov. This method is based on determining the location of a given point relative to all half-spaces that participate in the description of the computational domain. Then, the belonging of the point to the combinations of half-spaces is gradually determined, where the operations of intersection and negation of half-spaces are involved. It is this way of localizing points that gives the idea to obtain images of flat sections of three-dimensional regions by planes.

Let a three-dimensional domain consist of a combination of half-spaces bounded by planes and surfaces of the second order. Then the boundary of the region will consist of surfaces of the first and second order and is piecewise smooth. Further, let an arbitrary plane be given in three-dimensional space. It is required to build an image of the cross section of the figure with this plane. The following algorithm is proposed to solve this problem (The figures show the results of performing actions obtained by crossing the area shown in Fig.3 with the Z=0 plane).

1). The intersection lines of the first and second order surfaces involved in the description of the figure with the secant plane are determined separately. As a result, the coefficients of the general equation of lines of the first and second order (lines, circles, ellipses, parabolas, hyperbolas, etc.) are obtained.

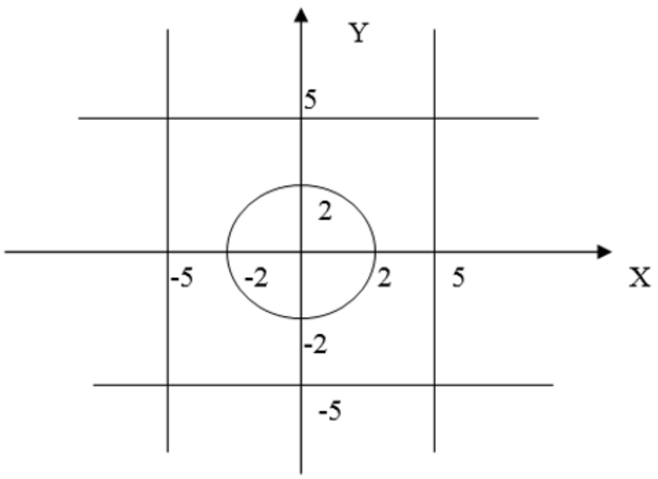


Figure 4. Schematic representation of the lines of intersection of the surfaces of the area with the secant plane

In the case of our example, the plane Z = 0 intersects the planes 1,2,3 and 4 in straight lines $x - 5 = 0$, $x + 5 = 0$, $y - 5 = 0$ and $y + 5 = 0$, respectively, it does not intersect with planes 5 and 6 at all, but with the surface $x^2 + y^2 + z^2 - 4 = 0$ intersects along the circle $x^2 + y^2 = 4$. Their schematic representation is shown in Fig.4.

2). On the secant plane, the rectangle containing all the obtained lines inside itself is determined (Fig.5).

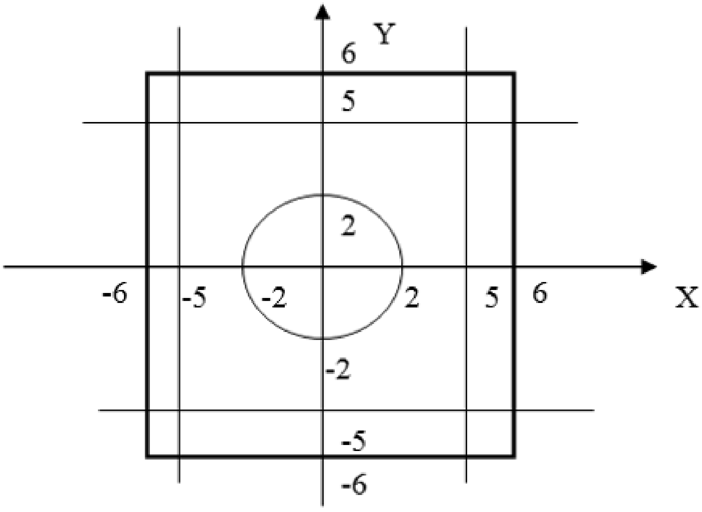


Figure 5. Schematic representation of the rectangle containing all the obtained lines inside itself

3). Using straight lines parallel to the sides of the rectangle, a uniform grid is constructed (Fig.6).

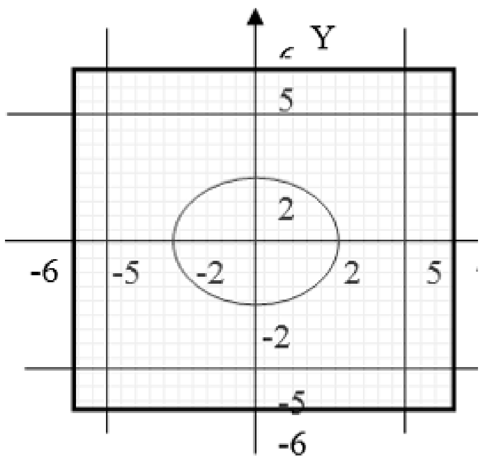


Figure 6. Constructing a grid on the fig 5.

4). The points of intersection of the grid lines with the lines of the first and second order obtained in point 1 of our algorithm are found. The result is a set of points lying inside the rectangle (Fig. 7).

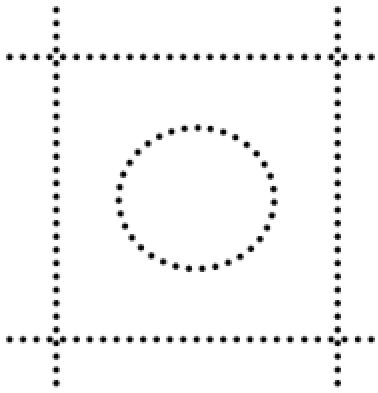
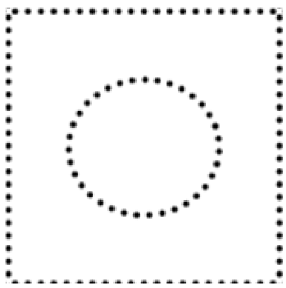


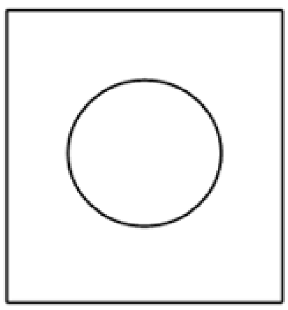
Figure 7. Schematic representation of the points where the grid lines intersect with the boundary lines of the specified area

5). Localization of all points of a given set relative to a given three-dimensional shape is carried out. As a result of localization, the location of each point relative to the boundary of a given area is determined, internal and external points are "thrown out", leaving only the boundary points (Fig.8).



Fugure 8. Schematic representation of the boundary points only

6). Take a set of only boundary points and connect their close neighboring points to each other with straight line segments and get the desired cross section (Fig.9).



Fugure 8. Schematic representation of the final cross section

The image obtained as a result of executing this algorithm is a section of a given three-dimensional shape with a given secant plane. The image quality depends on the step of the uniform grid constructed in clause 3 of the proposed algorithm. The smaller the grid pitch, the better the image quality on the computer display. The algorithm has been tested in the study of some applied problems and is quite suitable for creating modern electronic learning tools. But it should be noted that when constructing the central section of the cone using the proposed algorithm, there is a slight distortion of the vertex when the grid pitch exceeds a certain value. This distortion is due to the method of sequentially connecting the points of the set used in clause 6 of the algorithm proposed above.

The above method can theoretically be formulated as follows.

Let Ω be a three-dimensional bounded closed region whose boundary consists of pieces of P_i surfaces defined by their general equations

$$F_i(x,y,z)=0,$$

where $i = 1,2,3,...,I$ is the surface number according to some numbering, I is the total number of surfaces that make up the boundary of the region.

We introduce the following concepts for a given small positive number.

Definition 1. The set $V_{in} = \{(x,y,z) \in R^3: F(x,y,z) < - \varepsilon\}$ is called the interior of the surface F(x,y,z) = 0 with a given small ε accuracy.

Definition **2**. The set $V_{out} = \{(x,y,z) \in R^3: F(x,y,z) > \varepsilon\}$ is called the exterior of the surface F(x,y,z) = 0 with a given small ε accuracy.

Definition **3**. The set $V_b = \{(x,y,z) \in R^3: |F(x,y,z)| < \varepsilon\}$ is called the boundary of the surface F(x,y,z) = 0 with a given small ε accuracy.

It is known from set theory that any region with a piecewise smooth boundary can be formed from the sets V_{in} ,V_{out} and V_b using the intersection and complement operations. Therefore, the area Ω can be described as an ordered set D, the elements of which are the surface numbers (correspond to the interior of the surfaces together with the boundary), the numbers of surfaces with a minus sign (correspond to the exterior of surfaces without a boundary) and special signs that define the boundaries of various subdomains and play the role of parentheses in the description.

3. The concept **of** localization **of** points relative **to the** calculated area.

Let's now set an arbitrary point with coordinates (x_0,y_0,z_0). Substituting these coordinates into the left sides of the equation of surfaces, we define the set A_0 = {$a_1,a_2,a_3,\dots,a_I$}, the elements of which are determined by the formula:

$$a_i = \begin{cases} 0, if\ (x_0, y_0, z_0) \in V_b \\ 1, if\ (x_0, y_0, z_0) \in V_{in} \\ 2, if\ (x_0, y_0, z_0) \in V_{out} \end{cases}$$

Further, let L_1 – be the location of a point (x_0,y_0,z_0) relative to one surface, L_2 – relative to another. The position of the point (x_0,y_0,z_0) relative to the areas obtained as a result of the complement operation (denoted by $\overline{L_1}$ and $\overline{L_2}$) and the intersections (denoted by $L_1 \cap L_2$) are determined using the following table (see Table 1).

Table 1.

L_1	$\overline{L_1}$	L_2	$\overline{L_2}$	$L_1 \cap L_2$
0	0	0	0	0
0	0	1	2	0
0	0	2	1	2
1	2	0	0	0
1	2	1	2	1
1	2	2	1	2
2	1	0	0	2
2	1	1	2	2
2	1	2	1	2

As a result of the sequential formation of the region Ω according to the elements of the ordered set D, using the above table, an unambiguous localization of the point (x_0,y_0,z_0) relative to the region Ω is obtained.

Note that by introducing the concept of a shape (a separate section of the boundary consisting only of a part of one surface defined by its general equation), it is possible to significantly reduce the number of operations in the komputer implementation of this algorithm.

References

1. Vladimirov V.S. Equations of mathematical physics. – ed. 4-E. – M.: Science. The main editorial office of the physics and mathematics literature. 1981. -512s.

2. A.A.Samarsky. Introduction to the theory of difference schemes.-M.: Nauka. The main editorial office of the physics and mathematics literature. 1971.

3. Ivanshin P.N. And 23 The finite element method: a course of lectures: a textbook / P.N. Ivanshin. – Kazan: Kazan. Univ., 2013. – 120 p.

4. Ilyin V.P., Turakulov A.A. On integro-balanced approximations of three-dimensional boundary value problems. Novosibirsk, Preprint of the CC of the SB of the RAS. 1993.

I.M. Sobol. Numerical methods of Monte_Carlo. -M.: Nauka. The main editorial office of the physics and mathematics literature. 1973.

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